preprint from the Proceedings of the IEEE International Conference on E-Commerce, E-Technology, and E-Service
Taipei, Taiwan (March 29-31, 2004)

Quantitative Approach to Using E-Commerce Data to Monitor and Control the Performance of a Supply Chain

Trung T. Pham
DIcentral Corporation
tpham@DIcentral.com

Abstract

This paper presents a quantitative approach to using E-commerce data to measure, monitor, and control the performance of a supply chain. The performance is measured in terms of the turn-around time for a business transaction. This performance is monitored and correlated to a set of parameters so that the resulting mathematical relation is used as a model to design control measures to bring the performance to a desirable level. Computational algorithm is presented in the modeling process, and design of control measures is derived based on this model.

1. Introduction

A supply chain [1-4] is a set of suppliers selling their products or services to a buyer. An organization buying products or services from a supply chain normally prefers to maintain a certain degree of satisfaction from conducting business with these entities. While the overall satisfaction is often perceived as being subjective, there are quantitative parameters that can be used to support an objective assessment of the performance of a supply chain.

The performance of a supply chain [5-8] can be measured with different criteria, e.g., the supplier's ability to deliver a complete purchase order (in one shipment) within a specified timeframe. For organizations moving toward electronic commerce, the rate of adoption can be used during the transition phase (from using traditional paper legacy system to an electronic data interchange system). For an organization with a supply chain already using electronic data interchange, the performance is normally measured in terms of a supplier's ability to response to a purchase order in a timely manner.

Traditionally, the performance of a supply chain is manually assessed [9-12]. Then, some control measures [13-16] are intuitively planned for a manager to apply either to the people or to the process involved at the supplier's organization in order to bring the performance to a certain desired level. This manual assessment can be subjective and the control process can consequently be rendered ineffective. Furthermore, the relation between the control measures and the performance is often not clearly understood, thus preventing effective communications throughout an organization's managerial hierarchy.

This paper outlines a quantitative approach to objectively measure the performance of a supply chain in an electronic commercial environment. In this approach, the supply chain is modeled as a mathematical process. E-commerce data are used to extract observable output, and a set of related factors is proposed as internal variables driving the output of this process. The process is modeled as a linear formulation whose parameters are estimated based on the available observable data. The mathematical representation is used as a model to design and evaluate control measures available for a manager to apply to the supply chain.

The solution to this quantitative approach can be implemented in a decision support system to provide analysis between the supply chain performance and the environment's variables. The suggested control measures can often be seen as a concise rule-base replacement for an extensive knowledge-base expert system.

2. Supply Chain Performance

An organization trying to optimize its just-in-time inventory system [17-18] must rely on the response time of its suppliers when a purchase order is placed. This response time is critical for the optimization of inventory cost and uninterrupted operation period. These factors play major roles in retaining competitive advantages over competitors.

The response time of a supplier is measured as the time duration between the time a purchase order is placed and the time the corresponding invoice is received. Through the use of electronic data interchange, the period when these documents are in transit is practically reduced to fraction of a second, leaving the response time as a true indicator of how efficient a supplier is set up.

3. Supply Chain Modeling

A model of a supplier is a mathematical expression correlating the performance variable with other environment variables involved in a purchasing cycle. The correlation is specified by a set of parameters. These parameters are estimated based on available input (environment variables) and output (performance result) data.

A. System Variables

For illustrative purpose, a finite set of variables is defined in this section under the assertion that this set can be extended to customize to a user's preference and business environment. Let y be the output (performance) of a supplier measured as the time difference between an invoice and a purchase order. Let the vector u be the variable vector consisting of e-commerce environment variables such as the number of items in an order, the total purchase amount, the quantity for each item, the geographical distance between the supplier's warehouse and its main office, etc. For a more complicated e-commerce environment, this vector can be extended to contain a larger number of variables without affecting the parameter estimation procedure derived in the following subsections.

B. Linear Modeling

In this setting, the model is formulated to be a direct linear relation between the performance and the environment's variables:

$$y = c^{T} u, \tag{1}$$

where u is a column vector containing the environment's variables, y the performance, and c a constant column vector containing the system parameters that characterize the environment.

The modeling problem is simplified to be solving a set of linear equations. Let $Y = \{ y_1, y_2, \ldots, y_N \}$ be a set of performance, each associated with a particular purchase order. For each performance y_n, a set of system variables $U_n = \{ u_{n1}, u_{n2}, \ldots, u_{nM} \}$ is recorded. The problem of modeling is to find a set of parameters $C = \{ c_1, c_2, \ldots, c_M \}$ that satisfies the following equation

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} u_{1,1} & u_{1,2} & \cdots & u_{1,M} \\ u_{2,1} & u_{2,2} & & u_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ u_{N,1} & u_{N,2} & \cdots & u_{N,M} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_M \end{bmatrix}. \tag{2}$$

The matrix equation (2) above can be routinely solved if the dimensions are consistent, i.e., $N = M$, and the matrix U is non-singular. In this case, there exists several algorithms to solve this matrix linear equation, the most popular and easy one to understand is the process of row/column elimination [19-20]. The solution is simply solving for the inverse of a square matrix:

$$\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_M \end{bmatrix} = \begin{bmatrix} u_{1,1} & u_{1,2} & \cdots & u_{1,M} \\ u_{2,1} & u_{2,2} & & u_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ u_{N,1} & u_{N,2} & \cdots & u_{N,M} \end{bmatrix}^{-1} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}. \tag{3}$$

When the dimension N is not equal to M, the problem becomes more interesting: there might be an infinite number of solutions when $N < M$, and no solution at all when $N > M$. In the first case of $N < M$, additional constraints can be applied to ensure a unique solution, e.g., the norm of the solution vector C is minimized to ensure an optimal bounded-input-bounded-output relation. In this scenario, the problem is converted into a constrained optimization:

$$\min_{C} \sum_{m=1}^{M} c_m^2 , \tag{4}$$

s.t.

$$\sum_{m=1}^{M} u_{n,m} c_m = y_m \qquad \text{for } n = 1, 2, \ldots, N. \tag{5}$$

This formulation yields a unique solution

$$c_p = \frac{\sum_{n=1}^{N} u_{n,p} y_p}{\sum_{n=1}^{N} \sum_{m=1}^{M} u_{n,m}^2} . \tag{6}$$

In the second case of $N > M$, there exists no solution (unless some linear equations can be shown as a linear combination of other linear equations, allowing a reduction in dimension toward an exact and unique solution). In general, approximation method is used to estimate a solution that minimizes the norm of the error function. The problem is reduced to an unconstrained optimization

$$\min_{C} \sum_{n=1}^{N} (y_n - \sum_{m=1}^{M} u_{n,m} c_m)^2 . \tag{7}$$

This formulation also yields a unique solution often known as the pseudo-inverse solution

$$C = (U^{T}U)^{-1} U^{T} Y. \tag{8}$$

The solution (8) exists under the specific condition that the resulting matrix $(U^{T}U)$ is non-singular.

C. Adaptive Solution

In an operational environment, the data are continuously accumulated, requiring the parameters to be constantly adjusted to reflect the newly collected data. In order to avoid repeating the calculation of a complete solution, an adaptive solution is derived to allow a less complicated calculation of the update that is simply added to the previous solution. This approach yields an adaptive solution [21-24].

The minimum norm solution in (6) can be arranged to become an adaptive solution as follows. Let $C^{(N)}$ be the solution calculated on the availability of N observations.

Supposed that an $(N+1)^{st}$ observation becomes available, the solution based on $(N+1)$ observations can be calculated as an update of the previous solution $C^{(N)}$ in the following formulation:

$$c_p^{(N+1)} = c_p^{(N)} + \frac{u_{N+1,p}\,y_p}{B^{(N)} + \sum_{m=1}^{M} u_{N+1,m}^2} - \frac{[\sum_{m=1}^{M} u_{N+1,m}^2]A^{(N)}}{B^{(N)}[B^{(N)} + \sum_{m=1}^{M} u_{N+1,m}^2]}, \quad (9)$$

where

$$A^{(N)} = \sum_{n=1}^{N} u_{n,p}\,y_p \,, \quad \text{and} \quad B^{(N)} = \sum_{n=1}^{N}\sum_{m=1}^{M} u_{n,m}^2 \,. \quad (10)$$

The minimum error solution in (8) to the modeling problem (7) can be reformulated based on the following special property of an inverse of the sum of two matrices:

$$(\Xi + \alpha\beta^T)^{-1} = \Xi^{-1} - \Xi^{-1}\alpha[\beta^T\Xi^{-1}\alpha - I]\beta^T\Xi^{-1}. \quad (11)$$

With this property, the adaptive solution to the modeling problem (7) is derived and summarized as follow:

$$C^{(K+1)} = \Omega^{(K+1)}\,\zeta^{(K+1)}, \quad (12)$$

where

$$\Omega^{(K+1)} = \Omega^{(K)} - \Omega^{(K)}v^{(K+1)}[v^{(K+1)T}\Omega^{(K)}\,v^{(K+1)} - I]\,v^{(K+1)T}\Omega^{(K)}, \quad (13)$$

$$\zeta^{(K+1)} = \zeta^{(K)} + y^{(K+1)}\,v^{(K+1)}, \quad (14)$$

$$v^{(K+1)} = [u_{(K+1),1} \quad u_{(K+1),2} \quad \ldots \quad u_{(K+1),M}]^T. \quad (15)$$

Notice that these formulations for updates require significantly fewer calculations than repeating the complete solution when the number of observable data becomes large.

D. Numerical Examples

Let u_1 be the number of line items on a purchase order, u_2 be the total purchase order amount, and y be the turnaround time. The model is formulated as:

$$y = c_1u_1 + c_2u_2, \quad (16)$$

where the constants c_1 and c_2 characterize the behavior of a supplier.

For a set of N purchase orders, the performance data set $Y = \{ y_1, y_2, \ldots, y_N \}$ and variable sets $U_1 = \{ u_{1,1}, u_{2,1}, \ldots, u_{N,1} \}$ and $U_2 = \{ u_{1,2}, u_{2,2}, \ldots, u_{N,2} \}$ are observed. The constants c_1 and c_2 are estimated to reflect the correlation between the performance Y and the system variables in the modeling process. The linear relation in (2) becomes:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} u_{1,1} & u_{1,2} \\ u_{2,1} & u_{2,2} \\ \vdots & \vdots \\ u_{N,1} & u_{N,2} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}. \quad (17)$$

When $N > 2$, the constants c_1 and c_2 can be estimated according to Equation (8) as follows:

$$c_1 = v_{1,1} \sum_{n=1}^{N} u_{n,1}y_n + v_{1,2} \sum_{n=1}^{N} u_{n,2}y_n \,, \quad (18)$$

$$c_2 = v_{2,1} \sum_{n=1}^{N} u_{n,1}y_n + v_{2,2} \sum_{n=1}^{N} u_{n,2}y_n \,, \quad (19)$$

where the constants $v_{1,1}$, $v_{1,2}$, $v_{2,1}$, and $v_{2,2}$ are:

$$v_{1,1} = \frac{\sum_{n=1}^{N} u_{n,2}^2}{\sum_{n=1}^{N} u_{n,1}^2 \sum_{n=1}^{N} u_{n,2}^2 - \left[\sum_{n=1}^{N} u_{n,1}u_{n,2}\right]^2}, \quad (20)$$

$$v_{2,1} = -\frac{\sum_{n=1}^{N} u_{n,1}u_{n,2}}{\sum_{n=1}^{N} u_{n,1}^2 \sum_{n=1}^{N} u_{n,2}^2 - \left[\sum_{n=1}^{N} u_{n,1}u_{n,2}\right]^2}, \quad (21)$$

$$v_{1,2} = -\frac{\sum_{n=1}^{N} u_{n,1}u_{n,2}}{\sum_{n=1}^{N} u_{n,1}^2 \sum_{n=1}^{N} u_{n,2}^2 - \left[\sum_{n=1}^{N} u_{n,1}u_{n,2}\right]^2}, \quad (22)$$

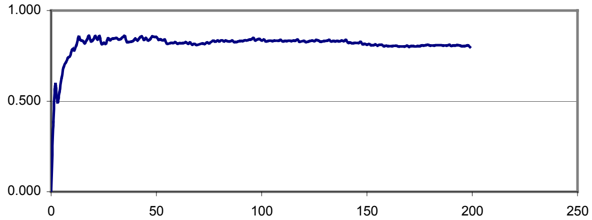
$$v_{2,2} = \frac{\sum_{n=1}^{N} u_{n,1}^2}{\sum_{n=1}^{N} u_{n,1}^2 \sum_{n=1}^{N} u_{n,2}^2 - \left[\sum_{n=1}^{N} u_{n,1}u_{n,2}\right]^2}. \quad (23)$$

The numerical simulation consists of three parts: creating a profile of simulated data Y and U based on some given constants c_1 and c_2 using equation (16), using the data profiles to estimate the two constants c_1 and c_2, and comparing the calculated constants c_1 and c_2 with the original constants used in the creation of simulated data Y and U.

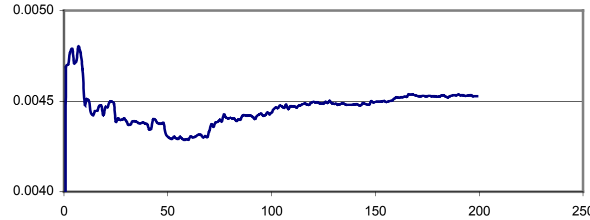
Let $c_1 = 0.5$ and $c_2 = 0.005$, and let the values set U be randomly created. Then, the performance set Y is calculated using equation (16). Figures 1-c and 1-d display the values of u_1 (number of line items in a purchase order) and u_2 (total purchase order amount). Figure 1-e shows the simulated performance y (turnaround time). There are 200 sample purchase orders generated in this example.

In a simulated situation, only the profiles in Figures 1-c, 1-d, and 1-e are considered observable data. The objective of the modeling process is to use these observable data to calculate the constants c_1 and c_2. Formulas (18) and (19) are used to estimate these constants, and the estimations are plotted in Figures 1-a and 1-b.

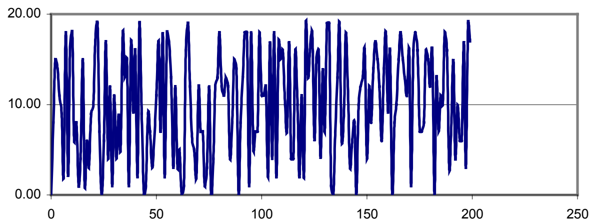
The estimated constants c_1 and c_2 are calculated according to a data set that increases one point at a time. They can be seen to be converging after about 100 data points. The convergence can be visually observed in Figures 1-a and 1-b to be asymptotical. These estimated constants are then used to calculate the model's value of y. This model's value of y is compared with the simulated value y created earlier. The squared norm of the error is



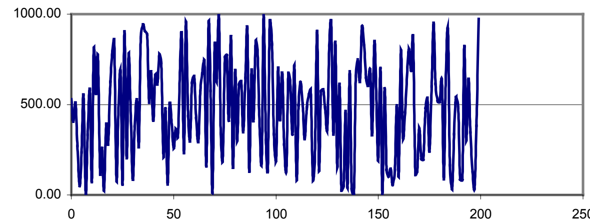
(a) Estimation of c_1 (true value = 0.5) as the function of N sample data points.



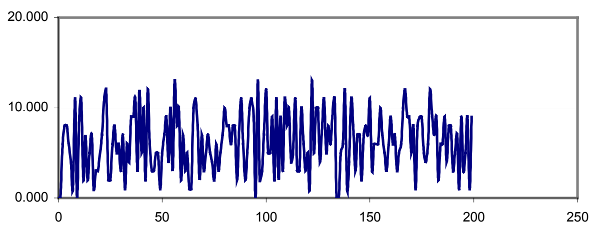
(b) Estimation of c_2 (true value = 0.005) as the function of N sample data points.



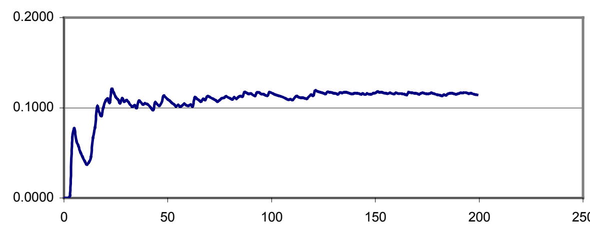
(c) Observable input data u_1 (number of line items).



(d) Observable input data u_2 (total purchase order amount).



(e) Observable output data y (response time in hours).



(f) Estimation error ε as the function of N sample data points.

Figure 1. Estimation of parameters c_1 and c_2 based on the model $y = c_1 u_1 + c_2 u_2$ where $c_1 = 0.5$ and $c_2 = 0.005$.

plotted in Figure 1-f. Here, the error is also visually observed as converging asymptotically.

4. Control of Supply Chain Performance

A control measure applied to a supply chain can be designed based on a mathematical model that describes the chain's behavior. Recall that in the previous section, a mathematical model was set up to correlate the performance with certain variables.

For a linear model with mathematical description

$$y = [\, c_1\, c_2 \ldots c_M \,] \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_M \end{bmatrix}, \tag{24}$$

the control variables $u_1, u_2, \ldots, u_M$ can be designed according to the values of the model's constant parameters $c_1, c_2, \ldots, c_M$ as follows.

The objective of designing a control measure of a system is to minimize a cost function associating with that system. The optimization problem is set up in the form

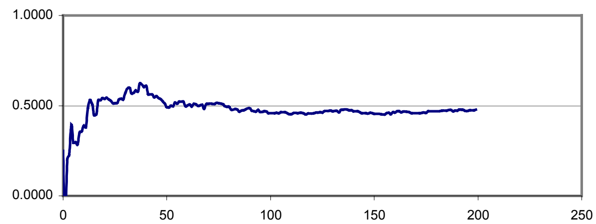
$$\min_{u_1,\cdots,u_M} y\,, \tag{25}$$

s.t.

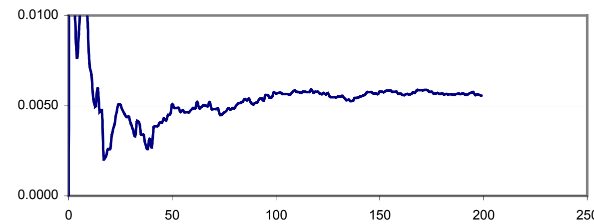
$$y = \sum_{m=1}^{M} c_m u_m\,, \tag{26}$$

$$u_{m,\min} \le u_m \le u_{m,\max}, \tag{27}$$

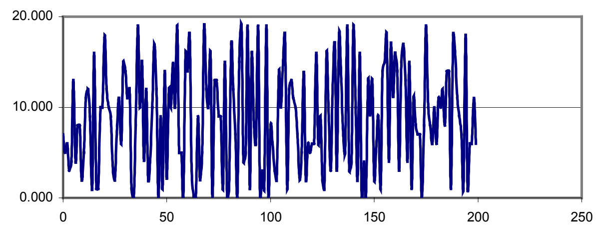
where y is the system output, and the constraint $y = Cu$ is the model of the system behavior. Notice that this optimization problem is in the special form of a linear programming problem with equality constraints (instead of inequality constraint) and each variable is bounded in a finite interval. This special form allows a closed form solution calculated directly in one step (instead of the traditional search along a boundary of the constraints in a linear programming problem). The solution represents a



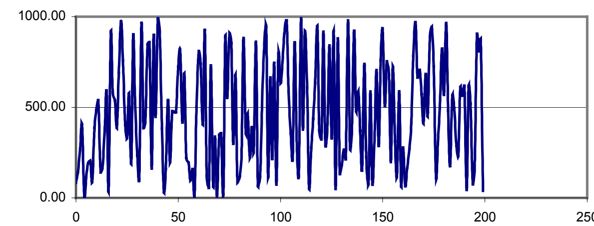
(a) Estimation of c_1 (true value = 0.5) as the function of N sample data points.



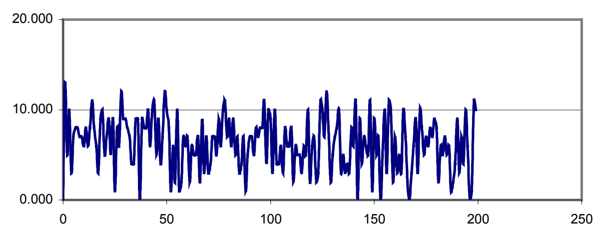
(b) Estimation of c_2 (true value = 0.005) as the function of N sample data points.



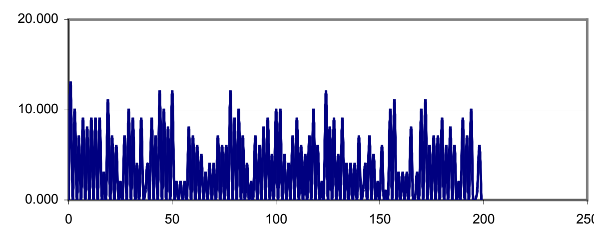
(c) Observable input data u_1 (number of line items).



(d) Observable input data u_2 (total purchase order amount).



(e) Observable output data y (response time in hours).



(f) Closed loop output data (with control measure).

Figure 2. Simulated results of the analytical control measure listed in Equation (28).

lowest possible point of a flat surface in a multi-dimensional space.

If the constant c_p is negative, then increasing the value of the control variable u_p will improve the performance while decreasing it will deteriorate the performance. Vice versa, if the constant c_p is positive, then decreasing the value of the control variable u_p will improve the performance while increasing it will deteriorate the performance. These rules can be summarized as follows. Let the range $[u_{p,\min}, u_{p,\max}]$ be available for each control variable u_p, then

$$u_p = \begin{cases} u_{p,\max} & \text{if } c_p < 0 \textit{ and } u_{p,\min} > 0, \\ u_{p,\min} & \text{if } c_p > 0 \textit{ and } u_{p,\min} > 0, \\ u_{p,\max} & \text{if } c_p < 0 \textit{ and } u_{p,\max} < 0, \\ u_{p,\min} & \text{if } c_p > 0 \textit{ and } u_{p,\max} < 0. \end{cases} \quad (28)$$

Notice that there are cases when the control variable is calculated to be of some negative value and the reality of a supply chain system is that the value of that particular variable must be a positive non-negative number, e.g., the number of line items in a purchase order. In this case, the variable becomes a non-controlling variable due to its limitation. The system is then considered uncontrollable from that input variable.

Figure 2 illustrates the application of the control measure in Equation (28) in numerical simulation. Figures 2-a through 2-e show the simulated data generated in the same manner described in the previous Section. Using these simulated data, appropriate control actions are designed and fed to the simulation equation $y = c_1u_1 + c_2u_2$, the results are plotted in Figure 2-f. It can be observed visually that the performance profile with applied control measures (shown in Figure 2-f) is significantly better than the performance profile without control measures (shown in Figure 2-e). The response time seems to be improved to be about half the response time without control measures.

5. Conclusion

It was shown that in order to improve the manual control process that reduces the response time of a supplier, mathematical model can be used to correlate the cause and effect of the related factors. This mathematical model was based on objective measurements of the actual system, eliminating the subjective assessment of a performance. The model also served as the basis to provide analysis and explanation of a performance profile. This basis, when expressed in mathematical equations, can be used to devise control measures aiming at bringing the performance to a desired (better) level.

References

[1] D. J. Bowersox, D. J. Closs, M. B. Cooper. *Supply chain logistics management*. Boston, MA: McGraw-Hill (2002).

[2] R. Cooper, R. Slagmulder. *Supply chain development for the lean enterprise: interorganizational cost management*. Montvale, NJ: IMA Foundation for Applied Research (1999).

[3] W. C. Benton. "Supply chain partnerships: opportunities for operations research," *European Journal of Operation Research*, vol. 101 (1997), pp 419-429.

[4] M. L. Fisher. "What is the right supply chain for your product?" *Harvard Business Review* (1997), pp 105-116.

[5] S. Chopra, P. Meindl. *Supply chain management: strategy, planning, and operation*. Upper Saddle River, NJ: Prentice Hall (2001).

[6] A. J. Berger and J. L. Gattorna. *Supply chain cybermastery: building high performance supply chains of the future*. Burlington, VT: Gower (2001).

[7] C. Muralidhara, N. Anantharaman, and S.G. Deshmukh, "A multi-criteria group decision making model for supplier rating," *The Journal of Supply Chain Management*, vol. 38, no. 4 (Fall 2002), pp. 22-33.

[8] R. Narasimhan, S. Talluri, and D. Mendez. "Supplier evaluation and rationalization via data envelopment analysis: an empirical examination," *The Journal of Supply Chain Management*, vol. 37, no. 3 (Summer 2001), pp. 28-37.

[9] R. A. Berk (ed.). *Performance assessment: methods and applications*. Baltimore, MD: Johns Hopkins University Press (1986).

[10] J. Holloway, J. Lewis and G. Mallory (eds.). *Performance measurement and evaluation*. London, UK: Sage Publications (1995).

[11] S.V. Walton, and A.S. Marucheck. "The relationship between EDI and supplier reliability," *International Journal of Purchasing and Materials Management*, vol. 33, no. 3 (1997) pp. 30-35.

[12] K. C. Tan, V. R. Kannan, and R. B. Handfield. "Supply chain management: supplier performance and firm performance," *Internaltional Journal of Purchasing and Materials Management*, vol 34, no. 3 (1998) pp 2-9.

[13] C. C. Poirier. *The supply chain manager's problem-solver: maximizing the value of collaboration and technology*. Boca Raton, FL: St. Lucie Press (2003).

[14] E. Frazelle. *Supply chain strategy: the logistics of supply chain management*. New York, NY: McGraw-Hill (2002).

[15] S. Prasad, J. Tata and J. Motwani. "International supply chain management: learning & evolving networks," *Global Journal of Flexible Systems Management* Vol. 2, No. 2 (2001) pp 31-36.

[16] C. Scott, and R. Westbrook. "New strategic tools for supply chain management," *International Journal of Physical Distribution and Logistics Management*, vol. 21, no. 1 (1991) pp 13-22.

[17] R. T. Lubben. *Just-in-time manufacturing: an aggressive manufacturing strategy*. New York, NY: McGraw-Hill (1988).

[18] S. E. Larson. *Inventory systems and controls handbook*. Englewood Cliffs, NJ: Prentice-Hall (1976).

[19] J. J. Dongarra. *LINPACK: users' guide*. Philadelphia, PA: Society for Industrial and Applied Mathematics (1979).

[20] C. Meyer. *Matrix analysis and applied linear algebra*. Philadelphia, PA: Society for Industrial and Applied Mathematics (2000).

[21] A. Benveniste, M. Métivier, P. Priouret. *Adaptive algorithms and stochastic approximations* (translated by Stephen S. Wilson). New York, NY: Springer-Verlag (1990).

[22] K. Kreith, D. Chakerian. *Iterative algebra and dynamic modeling: a curriculum for the third millennium*. New York, NY: Springer (1999).

[23] C. T. Kelley. *Iterative methods for linear and nonlinear equations*. Philadelphia, PA: Society for Industrial and Applied Mathematics (1995).

[24] G. Tao. *Adaptive control design and analysis*. Hoboken, NJ: Wiley-Interscience (2003).