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Adaptive Fuzzy Approach to Estimate Supplier's Competitiveness in Open e-Bidding

Trung T. Pham
DIcentral Corporation
tpham@DIcentral.com

Abstract

This paper presents an adaptive fuzzy approach to estimate the competition of the suppliers in an open electronic bidding. The competition is formulated as the supply curve relating pricing against requested quantity. A set of supply curves, each from a competitor, is formulated into fuzzy expression to allow flexibility for a supplier to prepare an optimal pricing policy that deals with the fact that other competitors are also adapting at some unknown rate. The pricing policy is formulated into a mathematical expression that is easily adjusted as new information on the competitors becomes available. A revision of pricing policy for a supplier to compete on the pricing basis while maintaining an optimal profit margin is derived as an application. Computer simulations are provided to demonstrate the workability of this approach.

1. Introduction

An open electronic bidding [1-4] is the process of soliciting and selecting a best quote from a set of suppliers. The results of this open electronic bidding are posted after the selection for all to see to ensure that the selection process is impartial. In order to improve their chances, suppliers often study this result and revise their pricing policy against the competitions.

The emergence of web services through the Internet has made electronic bidding a much faster process and the resulting data more readily available [5-6]. Thus, it is important for a supplier to be able to revise their pricing policy to adjust to the competition for survival. The past practice of manually adjusting the pricing policy becomes tedious and time consuming in this fast pace of e-bidding activities. In order to survive the new trend of e-bidding, suppliers must be able to adjust their pricing policy at a much faster pace while maintaining the objective of optimizing their profits.

A supply curve is a plot showing the unit price that a supplier is willing to sell its product for when a certain number of units is requested [7-10]. This supply curve is often used to analyze the competitiveness of that supplier, or to match up against the demand curve [11-14] for determining an equilibrium of optimal price and optimal quantity for both the buyer and supplier. A supplier normally wants to know its competitors and their competitiveness to prepare its own pricing policy. A buyer normally wants to know its suppliers' competitiveness to schedule its buying pattern that optimizes the cost.

This paper proposes the use of fuzzy expression [15-19] to represent a supply curve. The fuzzy expression is of the form if-then statement, where the data are described in terms of fuzzy sets. The advantage of using fuzzy expression is the flexibility of data accuracy: the fuzzy expression covers a set of curves within some upper and lower bounds. This flexibility allows a broad treatment of a family of curves in a situation where the actual available information might not be sufficient to determine the exact relationship.

The fuzzy supply curve for each supplier will be continuously refined with the data released from an open electronic bidding. Adaptive algorithm is developed to avoid unnecessary repeating calculation of the same historical data every time a new additional data point becomes available. For a supplier to remain competitive, its pricing policy must reflect a strategy of beating its competition, i.e., it must design its pricing policy based on knowledge of its competitors' supply curves. This paper presents an optimization problem formulating a pricing policy based on a set of supply curves from a field of competitors. An optimal solution is presented by solving this problem under the internal cost constraint. Computer simulations are presented to demonstrate the workability of this new concept.

2. Fuzzy Supply Curve

A fuzzy expression can be used to describe a family of analytical curves. The popular fuzzy expression is of the if-then form:

$$\text{if } q \in S_n \text{ then } p \in S_m, \qquad (1)$$

where q is the quantity requested, and p the corresponding unit price of a product. The sets S_n and S_m are defined

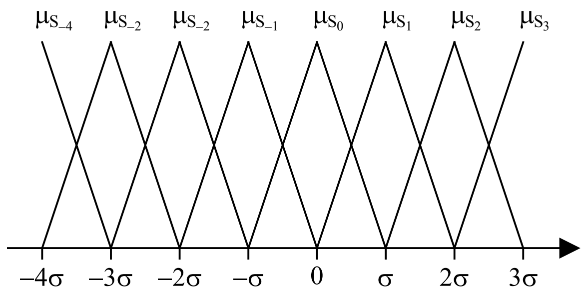

Figure 1. Triangular membership functions over equidistant intervals.

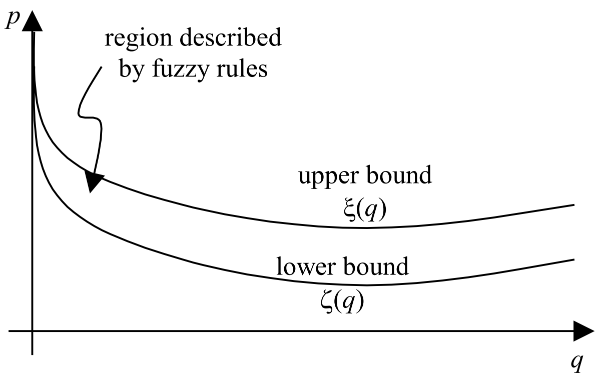

Figure 2. A typical class of supply curves bounded from above and below.

over some ranges $[q_{\min},q_{\max})$ and $[p_{\min},p_{\max})$, each associates with a given fuzzy membership function $\mu_{Sn}(q)$ or $\mu_{Sm}(p)$ that describes the fitness of the data in that set.

Specifically, when the fuzzy set is S_n is characterized by its integer index n, i.e., $S_n = [(n-1)\sigma,(n+1)\sigma)$ where the constant σ determines the width of that set, and its corresponding membership function $\mu_{Sn}(q)$ has a triangular shape (Figure 1), i.e.,

$$\mu_{Sn}(q) = \begin{cases} (q - n\sigma)/\sigma & \text{if } (n-1)\sigma \le q \le (n)\sigma, \\ (n\sigma + \sigma - q)/\sigma & \text{if } (n)\sigma \le q \le (n+1)\sigma, \\ 0 & \text{else,} \end{cases} \quad (2)$$

then the fuzzy relation in (1) can be shown to be bounded from above and from below. For a typical supply curve where unit price generally goes down as the quantity goes up, the fuzzy relation in (1) can be specifically defined as:

$$\text{if } p(q) \in S_n \text{ and } c(q) \in S_m, \quad \text{then } p(q+1) \in S_{-an+bm}, \quad (3)$$

where the constants a and b are the characteristic of the fuzzy supply curve, and the functions $p(q)$ is the unit price at the quantity q, $c(q)$ the basic unit cost of producing a product at the quantity q. These constants a and b along with the width σ of the indexed fuzzy sets S_n for n = 0, 1, 2, … determine the upper bound $\xi(\cdot)$ and lower bound $\zeta(\cdot)$ as follows:

$$\zeta(q) \le p(q) \le \xi(q), \quad (4)$$

where

$$\zeta(q+1) = -a\zeta(q) + bc(q) - \gamma \quad (5)$$

$$\xi(q+1) = -a\xi(q) + bc(q) + \gamma \quad (6)$$

$$\gamma = (\,|\,a\,| + |\,b\,|\,)\,\sigma. \quad (7)$$

These equations also depend on the initial conditions:

$$\zeta(0) = \zeta_0, \quad (8)$$

$$\xi(0) = \xi_0. \quad (9)$$

Most of the time, the initial conditions can be set to $\zeta_0 = \xi_0 = p_0$ for both bounds can start at the actual initial condition p_0. When the constant a is positive, the upper bound $\xi(\cdot)$ and lower bound $\zeta(\cdot)$ will curve down from their initial condition and converge to a constant (Figure 2). This is a typical behavior of a supply curve. The price will go down as the requested quantity goes up. The price will level off at some saturation point where the supplier cannot sell the product any cheaper while maintaining some profit.

3. Adaptive Estimation of Supply Curves

In this section, an algorithm is derived to estimate the fuzzy supply curve based on the information collected from results of electronic biddings. In this setting, each supplier is assumed to have a pricing policy on a particular product. This pricing policy is reflected by the supply curve described in (1). Each time a bidding result is released, the following information is available: price $p_S(q)$ at a given quantity q provided by supplier S. From a buyer perspective, the cost $c(q)$ is always available. From a supplier perspective, it is common to assume that the cost is similar from one supplier to another supplier and therefore the entity $c(q)$ can be assumed known.

A. Problem Formulation

The problem of modeling a pricing curve is to assume a mathematical expression representing the curve where a set of parameters is used to vary the characteristics of that curve. These parameters are estimated based on the observed data in a way that minimizes the error between the mathematical model and the actual data.

In the context of this paper, the mathematical expression is assumed of the form given in (3) where the constants a and b are the characteristics of the curve the expression represents. Assuming a profile of data $\{ (p(q_n), u(q_n), q_n) \mid n = 0, 1, 2, \dots \}$ is available. The problem of modeling the supply curve can be formulated into a constrained optimization problem minimizing the error function under the constraints of the boundaries of the fuzzy model:

$$\min_{a,b} \|\varepsilon\|^2, \quad (9)$$

s.t.

$$\varepsilon(q) = \max(\,|\zeta(q) - p(q)|, |\xi(q) - p(q)|\,), \quad (10)$$

$$\zeta(q+1) = -a\zeta(q) + bc(q) - \gamma, \quad (11)$$
$$\xi(q+1) = -a\xi(q) + bc(q) + \gamma, \quad (12)$$
$$\gamma = (| a | + | b |) \sigma. \quad (13)$$

Specifically, given an input signal $c(q)$ for $q = 1, 2, \ldots$, and the profile $p(q)$ for a finite set $q \in \{ q_1, q_2, \ldots, q_N \}$, estimate the parameters a and b such that the norm of the error ε in (9) is minimized. In the context of this paper, an l_2 norm is used, i.e.,

$$\| \varepsilon \|^2 = \sum_{n=1}^{N} \varepsilon_n^2 . \quad (14)$$

Notice that equation (10) defines the error function in a way that, when minimized, is equivalent to putting the data point $p(q)$ as close to the midpoint between the upper bound and the lower bound as possible. The midpoint of the upper bound and lower bound is represented by:

$$x(q) = \frac{1}{2} [\xi(q) + \zeta(q)]. \quad (15)$$

Substituting equations (11) and (12) into (15), the midpoint function $x(q)$ is obtained as:

$$x(q+1) = -ax(q) + bc(q). \quad (16)$$

In summary, the problem of estimating the parameters a and b for the fuzzy supply curve can be stated in term of an optimization problem:

$$\min_{a,b} \sum_{n=1}^{N} [x(q_n) - p(q_n)]^2 , \quad (17)$$

s.t.

$$x(q_n+1) = -ax(q_n) + bc(q_n), \quad (18)$$

for a given set of $c(q_n)$ for $n = 1, 2, \ldots, N$.

B. Optimal Solution

The optimization problem in (17) and (18) can be easily solved if the data is available for the range $q \in \{ 1, 2, \ldots, N \}$. In this case, the problem is converted into an unconstrained optimization problem. Let $x(q_n) = x_n$, $p(q_n) = p_n$, and $c(q_n) = c_n$ for shorter notation. Furthermore, let

$$y_n = x_n - p_n, \quad (19)$$

then,

$$y_{n+1} = x_{n+1} - p_{n+1}, \quad (20)$$
$$= -ax_n + bc_n - p_{n+1}. \quad (21)$$

Substitute (19) and (21) into (17), the constrained optimization problem in (17) and (18) becomes an unconstrained optimization problem:

$$\min_{a,b} \sum_{n=1}^{N} [-ax_{n-1} + bc_{n-1} - p_n]^2 . \quad (22)$$

The solution to this problem can be easily obtained by solving the following two equations

$$\frac{\partial}{\partial a} \sum_{n=1}^{N} [-ax_{n-1} + bc_{n-1} - p_n]^2 = 0 , \quad (23)$$

$$\frac{\partial}{\partial b} \sum_{n=1}^{N} [-ax_{n-1} + bc_{n-1} - p_n]^2 = 0 . \quad (24)$$

After applying the partial derivatives to the two equations (23) and (24), the results can be rewritten in form of a linear equation

$$\begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}, \quad (25)$$

where

$$\lambda_{11} = \sum_{n=1}^{N} x_{n-1}^2 , \quad (26)$$

$$\lambda_{12} = -\sum_{n=1}^{N} x_{n-1} c_{n-1} , \quad (27)$$

$$\lambda_{21} = -\sum_{n=1}^{N} x_{n-1} c_{n-1} , \quad (28)$$

$$\lambda_{22} = \sum_{n=1}^{N} c_{n-1}^2 , \quad (29)$$

$$\delta_1 = \sum_{n=1}^{N} x_{n-1} p_n , \quad (30)$$

$$\delta_2 = -\sum_{n=1}^{N} c_{n-1} p_n . \quad (31)$$

The solution for equation (25) is easily obtained in closed form as:

$$a = \frac{\lambda_{22}\delta_1 - \lambda_{21}\delta_2}{\lambda_{11}\lambda_{22} - \lambda_{21}\lambda_{12}}, \quad (32)$$

$$b = \frac{-\lambda_{12}\delta_1 + \lambda_{11}\delta_2}{\lambda_{11}\lambda_{22} - \lambda_{21}\lambda_{12}}, \quad (33)$$

with λ_{11}, λ_{12}, λ_{21}, λ_{22}, δ_1, and δ_2, defined in (26-29).

Notice that the unconstrained optimization problem in (22) is of the quadratic form, therefore, a unique solution exists, and is given in closed form in (32) and (33).

However, in real life practice, the optimization problem in (17) and (18) rarely has data available in the range $q \in \{ 1, 2, \ldots, N \}$. Instead, the data are available according to what is released and will appear random. In this case, let the range $q \in \{ q_1, q_2, \ldots, q_N \}$, where $q_n < q_m$ for $n < m$. The problem cannot be solved by simple substitution to reduce to an unconstrained optimization problem because equation (18) cannot be immediately used for the next available data point. Instead, equation (18) must be iterated repeatedly until the next point is obtained before the substitution can be applied. These iterations make the problem more complicated and more difficult to solve.

There are two approaches to solving this problem. The first approach is to fill in the data between points q_n and q_{n+1} with simple linear interpolation so that the data appear to be available for a continuous range described earlier. The solution in (32) and (33) can be used for this approach. Given any pair $p(q_n)$ and $p(q_{n+1})$, the following linear interpolation formula is used for this approach:

$$p(q_n+m) = p(q_n) + \frac{m[p(q_{n+1}) - p(q_n)]}{q_{n+1} - q_n}, \quad (34)$$

for m = 1, 2, …, $(q_{n+1}-q_n)$. Similarly, the interpolation formula above can be applied to calculate $x(q_n+m)$ and $c(q_n+m)$ to complete the data gaps. Notice that there are different interpolation formulas that can be used. The linear interpolation is the simplest and fastest, and is given as illustration purpose in this paper. For real life practice, one can choose an interpolation scheme that smoothes out the continuity in the sequence.

The second approach is to rewrite equation (18) into a form that computes the next point q_{n+1} based on q_n as follow:

$$x(q_{n+1}) = -ax(q_{n+1}-1) + bc(q_{n+1}-1), \quad (35)$$

which, can be substituted into itself repeatedly until the right hand side of the equation only contains $x(q_n)$:

$$x(q_{n+1}) = (-a)^{q_{n+1}-q_n} x(q_n) + \sum_{m=1}^{q_{n+1}-q_n} (-a)^{m-1} bc(q_{n+1}-m). \quad (36)$$

Equation (36) is a nonlinear function of the variables a and b. Any solution to the optimization problem with nonlinear constraints normally requires numerical approach to iterate a nominal solution toward an optimal one. This iterative approach is slow and only provides a numerical solution that might not be unique. For these reasons, the second approach is rather a research topic than a practical solution at this timeframe.

B. Adaptive Algorithm

A solution that involves a set of data collected over time is more efficiently calculated if it is updated each time a new data point is available instead of the solution being recalculated with all available data points collected over time.

In this section, the solution derived in the previous section is reformulated into an adaptive formula. Assuming N data points are available and a solution is already calculated based on these N data points. If a new data point is made available after this solution is calculated, then, a new solution can be calculated by repeating the procedure with $(N+1)$ data points. However, this procedure is tedious and wasting computational resources. An adaptive algorithm is the reformulation of a solution into two parts: one part involves the existing solution based on N data point, and another part involves some update of the existing solution with the new $(N+1)^{st}$ data point. In this concept, let $a^{(N)}$ and $b^{(N)}$ be the solution based on an existing set of N data points. With a new $(N+1)^{st}$ data point, the solution $a^{(N+1)}$ and $b^{(N+1)}$ can be calculated using an adaptive algorithm as:

$$a^{(N+1)} = f_1(a^{(N)}) + g_1(d_{N+1}), \quad (37)$$

$$b^{(N+1)} = f_2(b^{(N)}) + g_2(d_{N+1}), \quad (38)$$

where d_{N+1} is the newly available data point.

Equations (32) and (33) are the regular solution for the parameters of the supply curve. Substituting equations (26-31) into (32) and (33), the solution shows repeated use of the data set of N data points:

$$a^{(N)} = \frac{\sum_{n=1}^{N} c_{n-1}^2 \sum_{n=1}^{N} x_{n-1} p_n - \sum_{n=1}^{N} x_{n-1} c_{n-1} \sum_{n=1}^{N} c_{n-1} p_n}{\sum_{n=1}^{N} x_{n-1}^2 \sum_{n=1}^{N} c_{n-1}^2 - \left[\sum_{n=1}^{N} x_{n-1} c_{n-1}\right]^2}, \quad (39)$$

$$b^{(N)} = \frac{\sum_{n=1}^{N} x_{n-1} c_{n-1} \sum_{n=1}^{N} x_{n-1} p_n - \sum_{n=1}^{N} x_{n-1}^2 \sum_{n=1}^{N} c_{n-1} p_n}{\sum_{n=1}^{N} x_{n-1}^2 \sum_{n=1}^{N} c_{n-1}^2 - \left[\sum_{n=1}^{N} x_{n-1} c_{n-1}\right]^2}. \quad (40)$$

Substituting (39) and (40) into (37) and (38) respectively, the adaptive algorithm can be derived as follows:

$$a^{(N+1)} = \chi_1 a^{(N)} + g_1(d_{N+1}), \quad (41)$$

$$b^{(N+1)} = \chi_2 b^{(N)}) + g_2(d_{N+1}), \quad (42)$$

where the constants χ_1 and χ_2 are given as

$$\chi_1 = \frac{\sum_{n=1}^{N} x_{n-1}^2 \sum_{n=1}^{N} c_{n-1}^2 - \left[\sum_{n=1}^{N} x_{n-1} c_{n-1}\right]^2}{\sum_{n=1}^{N+1} x_{n-1}^2 \sum_{n=1}^{N+1} c_{n-1}^2 - \left[\sum_{n=1}^{N+1} x_{n-1} c_{n-1}\right]^2}, \quad (43)$$

$$\chi_2 = \chi_1, \quad (44)$$

and the functions $g_1(\cdot)$ and $g_2(\cdot)$ are given as

$$g_1(\cdot) = \chi_1[\, c_N^2 \sum_{n=1}^{N} x_{n-1} p_n + x_N p_{N+1} \sum_{n=1}^{N} c_n^2 - x_N c_N \sum_{n=1}^{N} c_{n-1} p_n - c_N p_{N+1} \sum_{n=1}^{N} x_{n-1} c_{n-1}\,], \quad (43)$$

$$g_2(\cdot) = \chi_2[\, x_N c_N \sum_{n=1}^{N} x_{n-1} p_n + x_N p_{N+1} \sum_{n=1}^{N} x_{n-1} c_{n-1} - x_N^2 \sum_{n=1}^{N} c_{n-1} p_N - c_N p_{N+1} \sum_{n=1}^{N} x_{n-1}^2\,]. \quad (44)$$

Notice that since the formulas in (32) and (33) are simple, each involves only 4 multiplications, 2 additions, and 1 division, it is common practice to derive the adaptive formulas for the constants in (26-31) where the formulation derivation is much simpler. Using the same technique in deriving (41) and (42), the adaptive algorithms for (26-31) can be obtained as follows:

$$\lambda_{11}^{(N+1)} = \lambda_{11}^{(N)} + x_N^2, \quad (45)$$

$$\lambda_{12}^{(N+1)} = \lambda_{12}^{(N)} - x_N c_N, \quad (46)$$

$$\lambda_{21}^{(N+1)} = \lambda_{21}^{(N)} - x_N c_N, \quad (47)$$

$$\lambda_{22}^{(N+1)} = \lambda_{22}^{(N)} + c_N^2 , \quad (48)$$

$$\delta_1^{(N+1)} = \delta_1^{(N)} + x_N p_{N+1}, \quad (49)$$

$$\delta_2^{(N+1)} = \delta_2^{(N)} - c_N p_{N+1}. \quad (50)$$

For each of the update in (45-50), the calculation only involves one multiplication and one addition. Even though the final calculation in (32) and (33) is repeated with no adaptive scheme, the intermediate variables are updated adaptively and the overall calculation scheme is quite efficient.

4. Optimal Pricing Policy

This section presents an application for possessing a set of supply curves from the suppliers for each individual product. Assuming that a supplier knows the field of its competitors, what each competitor is charging as unit price based on the ordered quantity, that supplier can correspondingly adjust its pricing policy to remain competitive in an open bidding environment.

A. Problem Formulation

Assuming a listing of pricing $p_n(q)$ for $n = 1, 2, \ldots, N$ competitors is available. It is important to reschedule one's pricing scheme to remain competitive. In this section, the problem is revising a pricing scheme is formulated into two steps: (i) evaluating competitor's pricing schemes and picking out the most competitive one, (ii) revising one's pricing scheme based on the best pricing scheme picked in the first step.

For a set of pricing schemes $p_n(q)$ for $n = 1, 2, \ldots, N$, the norms are calculated to evaluate the overall scheme. There are different norms that one can use, the most popular ones are the Laplacian norm (l_1 norm), Gaussian norm (l_2 norm), and l_∞ norm [20]. Let $\| \cdot \|_1$, $\| \cdot \|_2$, and $\| \cdot \|_\infty$ denote the three norms, then

$$\| p_n(q) \|_1 = \sum_{q=1}^{\infty} | p_n(q) | , \quad (51)$$

$$\| p_n(q) \|_2 = \sqrt{\sum_{q=1}^{\infty} | p_n(q) |^2} , \quad (52)$$

$$\| p_n(q) \|_\infty = \max_{q=1}^{\infty} | p_n(q) | . \quad (53)$$

Each norm places different emphasis on the distribution of the price according to the quantity. The Laplacian norm places even emphasis on all quantities, where the Gaussian places a little more emphasis toward the quantities with high price, and the l_∞ norm places emphasis only on the maximum price.

Once a best pricing is determined, it is used as the reference to set up the second part of the revising. Let $p^*(q)$ be the best pricing scheme, then, a supplier can use this as the reference in revising its own pricing scheme as close to this reference scheme as possible. This objective is formulated into an optimization problem as follows:

$$\min_{c(q)} \sum_{q=1}^{\infty} [x(q) - p^*(q)]^2 , \quad (54)$$

s.t.

$$x(q+1) = -ax(q) + bc(q), \quad (55)$$

This problem is sometimes considered the tracking problem, where cost $c(q)$ is designed to drive down the pricing scheme as much as possible to match the competitor's best pricing scheme $p^*(q)$. It is assumed that the parameters a and b are known in this supplier pricing model in (55). Additional constraint for c(q) can be applied to make the solution more practical, e.g.,

$$c(q) \geq 0. \quad (56)$$

B. Optimal Solution

The optimal problem in (54) and (55) can be easily solved by simplifying the objective function with a simple substitution:

$$z(q) = x(q) - p^*(q) \quad (57)$$

then, the problem can be rewritten as

$$\min_{c(q)} \sum_{q=1}^{\infty} z^2(q) , \quad (58)$$

s.t.

$$z(q+1) = -az(q) + bc(q) - p^*(q+1). \quad (59)$$

The constrained optimization problem in (58) and (59) is of the standard optimal control problem [21]. There exists a unique solution for this problem, and the solution is of the form:

$$c(q) = K(q)z(q), \quad (60)$$

where $K(q)$ satisfies the standard Riccati equation [22] associating with an optimal control problem [23-24]. Substitute equation (57) into (60), the final solution is obtained:

$$c(q) = K(q)[x(q) - p^*(q)], \quad (61)$$

5. Computer Simulations

In this section, computer simulations are presented to demonstrate the concept of using fuzzy logic expression to model a supply curve. Two numerical examples are listed in details to illustrate two cases when: (i) data are available for increasing quantities, and (ii) data are available only for some random quantities. The first case is presented as an example of an ideal modeling problem. The second case is presented as an example of a realistic situation often encountered in real-life scenarios.

A. Ideal Modeling Problem

In an ideal modeling problem, prices $p(q)$ are available for quantities $q = 1, 2, \ldots, N$. Each time a new price $p(N+1)$ becomes available, the model is adjusted to reflect the new information. This problem is simple and the

Table 1. Unit Prices Available for Example 1.

Quantity	Unit Price
1	$98.76
2	$78.56
3	$69.86
4	$57.63
5	$62.87
6	$53.99
7	$52.17
8	$58.87
9	$57.62
10	$55.18
11	$54.15
12	$52.01
13	$51.82
14	$56.51
15	$51.30
16	$48.43
17	$49.86
18	$50.70
19	$48.23
20	$50.71

Table 2. Unit Prices Available for Example 2.

Quantity	Unit Price
1	$102.22
6	$57.16
11	$53.45
44	$48.01
69	$55.55
72	$49.30
82	$49.98
87	$52.39
94	$49.10
112	$46.21
124	$54.73
126	$51.97
139	$53.64
188	$53.90
207	$51.38
300	$54.59
320	$49.13
337	$51.19
393	$53.65
399	$46.78

normal solution is easily derived in (32) and (33), and the corresponding adaptive solution is given in (45-50).

Let $P = \{ p_1, p_2, \ldots, p_N \}$ be a set of unit prices, set by a supplier, available for quantities $Q = \{ 1, 2, \ldots, N \}$, i.e.,

$$p(n) = p_n, \qquad \text{for } n = 1, 2, \ldots, N.$$

In this example, the data listed in Table 1 are available. This data set is generated with the formula:

$$p(n) = \frac{50n + 50}{n} + r,$$

where n is the quantity, r the random additive white noise routinely used (as a common practice) in computer simulation to illustrate the workability of a modeling solution in cases where data do not reflect an exact relationship. In this case, the unit price should start at $100.00 if the buyer buys only one unit, and gradually go down to about $50.00 as the buyer increases the quantity it wants to buy. The noise r, a real number ranging between −5 and +5, is added to introduce the unexpected cases when the supplier adjusted its prices irrationally, e.g., in response to give some one-time discount or one-time price increase. The noise r can also be used to introduce the inaccuracy in collecting pricing information.

It is obvious that the data in Table 1 cannot be modeled by an exact deterministic formula (see Figure 3). In this case, the traditional modeling approach will find a curve that yields the least error between the actual data and the model data. In fuzzy logic approach, the model is given in terms of an upper bound and lower bound that cover as many data points as possible. Thus, the fuzzy model allows a lot of flexibility in data accuracy and data sensitivity.

Given the these 20 points, the parameters a and b of the fuzzy model in Equations (32) and (33) are calculated:

$$a = \frac{\sum_{n=1}^{20} c_{n-1}^2 \sum_{n=1}^{20} p_{n-1} p_n - \sum_{n=1}^{20} p_{n-1} c_{n-1} \sum_{n=1}^{20} c_{n-1} p_n}{\sum_{n=1}^{20} p_{n-1}^2 \sum_{n=1}^{20} c_{n-1}^2 - \sum_{n=1}^{20} p_{n-1} c_{n-1} \sum_{n=1}^{20} p_{n-1} c_{n-1}} = 0.488,$$

$$b = \frac{\sum_{n=1}^{20} p_{n-1} c_{n-1} \sum_{n=1}^{20} p_{n-1} p_n - \sum_{n=1}^{20} c_{n-1}^2 \sum_{n=1}^{20} c_{n-1} p_n}{\sum_{n=1}^{20} p_{n-1}^2 \sum_{n=1}^{20} c_{n-1}^2 - \sum_{n=1}^{20} p_{n-1} c_{n-1} \sum_{n=1}^{20} p_{n-1} c_{n-1}} = -0.785.$$

The adaptive formulas in (45-50) for calculating these parameters can also be applied with data flowing in one piece at a time. The parameters a and b are plotted in Figures 4 and 5 to show how they evolves with new data. Figure 6 shows the upper bound and lower bound of the fuzzy model in (3) where with the parameters a and b just calculated above. It can be visually seen that the estimated parameters a and b converge as the number of data points increases.

B. Real-Life Modeling Problem

In this example, the same formula used to generate unit price data in Table 1 is used, but instead of generating data for quantities $q_1 = 1, q_2 = 2, \ldots, q_{20} = 20$, the unit price now is generated for quantities $q \in \{ q_1, q_2, \ldots, q_{20} \}$ where these quantities do not follow any

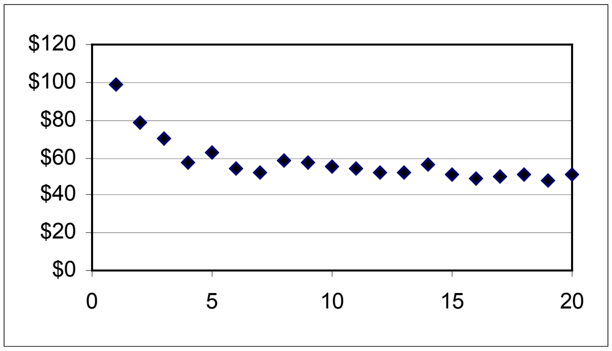

Figure 3. Simulated unit price data for Example 1.

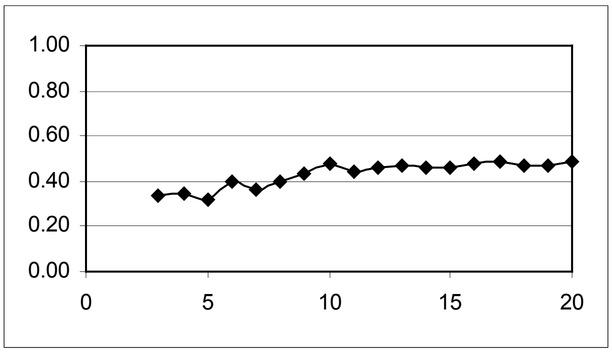

Figure 4. Adaptive estimation of parameters *a* in fuzzy model for Example 1.

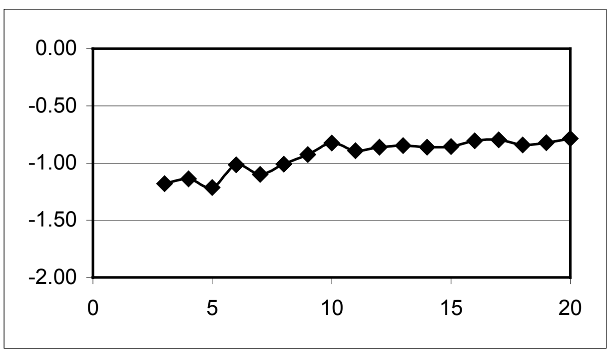

Figure 5. Adaptive estimation of parameters *b* in fuzzy model for Example 1.

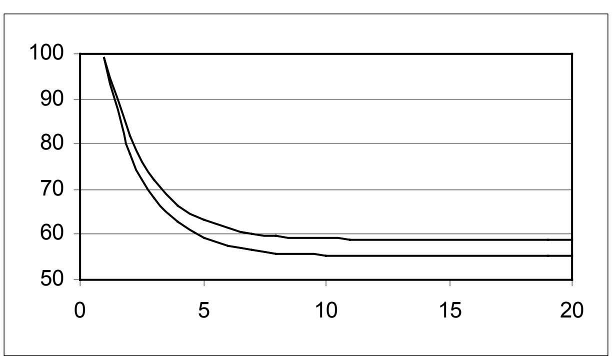

Figure 6. Fuzzy model with upper and lower bounds covering all data points in Example 1.

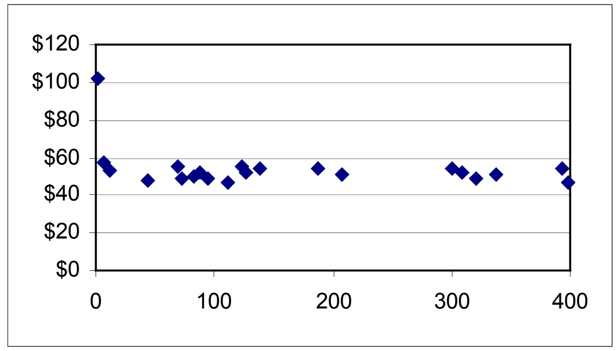

Figure 7. Simulated unit price data for Example 2.

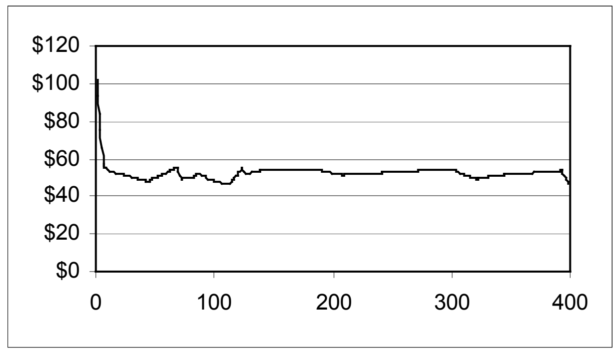

Figure 8. Interpolated unit price data for Example 2.

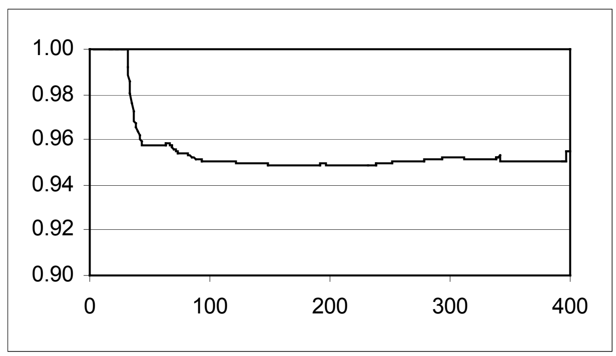

Figure 9. Adaptive estimation of parameters *a* in fuzzy model for Example 2.

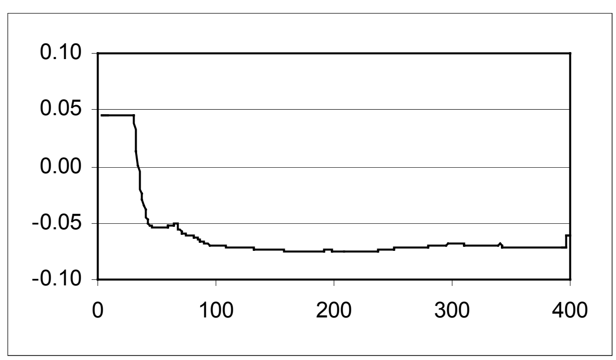

Figure 10. Adaptive estimation of parameters *b* in fuzzy model for Example 2.

example. First, 20 quantity values are generated randomly and ordered from small to large. Then, these quantity values are used to simulate the price $p(n)$ with the formula given in the previous example. Table 2 lists these values for comparison to the data in Table 1 to illustrate the difference between a real life situation and the ideal situation. Figure 7 shows these prices against the requested quantities. These prices are then used to interpolate the missing data between two non-consecutive quantities. The interpolated data are shown in Figure 8. Figures 9 and 10 show the adaptive estimation of the parameters a and b according to these 400 interpolated data points. It is understood that the fuzzy model will have the shape similar to that shown in Figure 6. Again, the values of a and b can be seen to be converging to some constants as the number of data points increases.

6. Conclusion

It has been shown that a family of supply curves can be described by a fuzzy if-then expression characterized by a set of parameters. These parameters are estimated based on the data released after an open bidding process to provide pricing policy from competitors. A supplier with this knowledge can appropriately adjust its pricing scheme to remain competitive in future biddings while maintaining consciousness of its cost structure. Algorithms for estimating the characteristic parameters of the supply curves and for adjusting a supplier's pricing scheme are derived from optimizing the estimating errors.

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